CAMBRIDGE
International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/14**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

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General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

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Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
^	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
✗	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
~~	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

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Annotation	Meaning
SF	Error in number of significant figures
✔	Correct
TE	Transcription error
XP	Correct answer from incorrect working

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\frac{1}{6}n(n+1)(2n+1)-n(n+1)+n$	**M1**	Substitutes all three correct formulae (two from MF19).
	$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$	**A1**	Must be in this form.
		2	
1(b)	**EITHER:** $\frac{1}{r^2-2r+1}-\frac{1}{r^2+2r+1}=\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}$		
	$\frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{2^2}-\frac{1}{4^2}+\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{4^2}-\frac{1}{6^2}+\ldots+\frac{1}{(n-2)^2}-\frac{1}{n^2}+\frac{1}{(n-1)^2}-\frac{1}{(n+1)^2}$	**(M1**	Writes enough correct <u>sections</u> of terms so that three of the four sections which appear in the final answer are seen.
		A1	All four correct sections correct and at least one cancellation pair included.
		A1	At least five complete n- value terms, either first three and last two, or first two and last three.
	$\frac{5}{4}-\frac{1}{n^2}-\frac{1}{(n+1)^2}$	**B1)**	OE WWW

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Question	Answer	Marks	Guidance
1(b)	**OR:** **Alternative method for 1(b)**		
	$\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots+\frac{1}{(n-2)^2}+\frac{1}{(n-1)^2}\right)-\left(\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\ldots+\frac{1}{n^2}+\frac{1}{(n+1)^2}\right)$	**(M1**	Writes enough correct <u>sections</u> of terms so that three of the four sections which appear in the final answer are seen.
		A1	All four correct sections correct and at least one cancellation pair included.
		A1	At least five complete n- value terms, either first three and last two, or first two and last three. WWW
	$\frac{5}{4}-\frac{1}{n^2}-\frac{1}{(n+1)^2}$	**B1)**	OE WWW
		4	
1(c)	$\frac{5}{4}$	**B1 FT**	FT *their* **1(b)** of similar form.
		1	

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Question	Answer	Marks	Guidance
2(a)	**EITHER:** $y = x^{-3} \Rightarrow x = y^{-\frac{1}{3}}$ $2y^{-\frac{4}{3}} + 3y^{-1} + y^{-\frac{1}{3}} + 1 = 0$	**(B1**	Correct substitution seen in the equation. You may see other forms including $2 + 3y^{\frac{1}{3}} + y + y^{\frac{4}{3}} = 0$ $y^{\frac{4}{3}} + 3y^{\frac{1}{3}} = -y - 2$
	$y(y+3)^3 = -(y+2)^3$	**M1**	Obtains an equation not involving roots or fractional powers from a correct substitution.
	$y(y^3 + 9y^2 + 27y + 27) = -(y^3 + 6y^2 + 12y + 8)$	**M1**	Expands powers of brackets.
	$y^4 + 10y^3 + 33y^2 + 39y + 8 = 0$	**A1**	Collects like terms. Answer must be an equation.
	$\alpha^{-3} + \beta^{-3} + \gamma^{-3} + \delta^{-3} = -10$	**B1FT)**	FT their answer equation.
	OR: **Alternative method for 2(a)**		
	$3x^3 + 1 = -2x^4 - x$ $(3x^3 + 1)^3 = -(2x^4 + x)^3$	**(M1**	Cube to make sure powers of x divide by 3
	$27x^9 + 27x^6 + 9x^3 + 1 = -8x^{12} - 12x^9 - 6x^6 - x^3$	**M1**	Expands powers of brackets.
	$27y^{-3} + 27y^{-2} + 9y^{-1} + 1 = -8y^{-4} - 12y^{-3} - 6y^{-2} - y^{-1}$	**B1**	Correct substitution seen in the equation.
	$y^4 + 10y^3 + 33y^2 + 39y + 8 = 0$	**A1**	Collects like terms. Answer must be an equation

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Question	Answer	Marks	Guidance
2(a)	$\alpha^{-3}+\beta^{-3}+\gamma^{-3}+\delta^{-3}=-10$	**B1FT)**	FT their answer equation.
		5	
2(b)	$\frac{1}{\alpha^6}+\frac{1}{\beta^6}+\frac{1}{\gamma^6}+\frac{1}{\delta^6}=10^2-2(33)$	**M1**	Relates $\frac{1}{\alpha^6}+\frac{1}{\beta^6}+\frac{1}{\gamma^6}+\frac{1}{\delta^6}$ to coefficients.
	34	**A1 FT**	FT their answer equation from 2(a) as long as it is a quartic equation.
		2	

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Question	Answer	Marks	Guidance
3(a)	$u_1 > 1$ [given]	**B1**	Base case seen.
	Assume that $u_k > 1$ for some positive integer k.	**B1**	States inductive hypothesis. Accept 'Let $u_k > 1$ be true' or 'Suppose $u_k > 1$.'
	EITHER: Then $u_{k+1} - 1 = \frac{u_k^2 + 2u_k + 2}{u_k + 4} - 1 = \frac{u_k^2 + 2u_k + 2 - (u_k + 4)}{u_k + 4} = \frac{u_k^2 + u_k - 2}{u_k + 4}$	**(M1**	Considers $u_{k+1} - 1$, and puts over common denominator. Numerator need not be simplified for this mark.
	$= \frac{(u_k - 1)(u_k + 2)}{u_k + 4} > 0$. Or $\frac{\left(u_k + \frac{1}{2}\right)^2 - \frac{1}{4} - 2}{(u_k + 4)} > 0$	**A1**	Factorises Or completes the square.
	Since $u_k > 1$ [by induction,] $u_n > 1$ for all positive [integers] n.	**A1)**	Must have previous M1 A1 and also explicitly connect the assumption $u_k > 1$ with their statement. No wrong working seen.
	OR: **Alternative for 3(a) M1 A1 A1marks**		
	Then $u_{k+1} - 1 = \frac{u_k^2 + 2u_k + 2}{u_k + 4} - 1 = \frac{u_k^2 + 2u_k + 2 - (u_k + 4)}{u_k + 4} = \frac{u_k^2 + u_k - 2}{u_k + 4}$	**(M1**	Considers $u_{k+1} - 1$, and puts over common denominator. Numerator need not be simplified for this mark.
	$u_k^2 > 1$ AND $u_k > 1$ so $u_k^2 + u_k - 2 > 1 + 1 - 2 = 0$	**A1**	Both must be seen.

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Question	Answer	Marks	Guidance
3(a)	Since $u_k > 1$ [by induction,] $u_n > 1$ for all positive [integers] n.	**A1)**	Must have previous M1 A1and also state $u_k > 1$ either in the conclusion or in the argument for the M1 A1. No wrong working seen. Allow for all positive integers [n].
		5	
3(b)	**EITHER:** $u_{n+1} - u_n = \frac{u_n^2 + 2u_n + 2 - u_n(u_n+4)}{u_n+4} = \frac{2(1-u_n)}{u_n+4}$	**(M1**	Considers $u_{n+1} - u_n$, or $u_n - u_{n+1}$, and puts over common denominator. Allow one numerical or sign error for M1.
		A1	
	So $u_n > 1 \Rightarrow u_{n+1} - u_n < 0$ $\Rightarrow u_{n+1} < u_n$	**A1)**	Uses $u_n > 1$. States result.
	OR: **Alternative for 3(b)**		
	$u_{n+1} - u_n = -2 + \frac{10}{u_n+4}$	**(M1**	$u_{n+1} - u_n = u_n - 2 + \frac{10}{u_n+4} - u_n$
	$u_n > 1,\ \ u_n + 4 > 5 \Rightarrow \frac{10}{u_n+4} < 2$	**A1**	
	$-2 + \frac{10}{u_n+4} < 0 \Rightarrow u_{n+1} - u_n < 0 \Rightarrow u_{n+1} < u_n$	**A1)**	States result.
		3	

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Question	Answer	Marks	Guidance
4(a)	$\begin{vmatrix}1 & k\\1 & -1\end{vmatrix} - k\begin{vmatrix}3 & k\\3 & -1\end{vmatrix} - \begin{vmatrix}3 & 1\\3 & 1\end{vmatrix} = -1-k-k(-3-3k)$ $=3k^2+2k-1=0$	**M1**	Finds determinant and
		A1	forms correct quadratic equation.
	$k=\frac{1}{3},-1$	**A1**	No method need be seen for solving quadratic equation.
		3	
4(b)	$\begin{pmatrix}0 & 1 & 0\\1 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & k & -1\\3 & 1 & k\\3 & 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\\1 & 0\end{pmatrix}=\begin{pmatrix}3 & 1 & k\\4 & k+1 & -2\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\\1 & 0\end{pmatrix}$	**M1 A1**	Multiplies two matrices correctly. Could see $\begin{pmatrix}0 & 1 & 0\\1 & 0 & 1\end{pmatrix}\begin{pmatrix}0 & -k\\3+k & -1\\2 & -1\end{pmatrix}$.
	$=\begin{pmatrix}k+3 & -1\\2 & -k-1\end{pmatrix}$	**A1**	AG. Some intermediate working must be shown. For first pairing of matrices must see 4 - 2 For second pairing must see 0 + 2
		3	

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Question	Answer	Marks	Guidance
4(c)	$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ 2x+y \end{pmatrix}$	**B1FT**	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$. Allow FT if left in terms of k.
	$2x+mx+c=m(x-mx-c)+c$	**M1 A1**	Uses $y=mx+c$ and $Y=mX+c$. M1 A0 if $c=0$ throughout.
	$\left(m^2+2\right)x+mc=0$	**A1**	c must be present here.
	$m^2+2=0 \Rightarrow$ no real values of m.	**A1**	Allow if c missing above. Accept 'no solutions'.
		5	

Question	Answer	Marks	Guidance
5(a)	$\overrightarrow{AB}=-2\mathbf{i}-\mathbf{j}-2\mathbf{k} \quad \overrightarrow{AC}=\mathbf{i}-\mathbf{j}-\mathbf{k} \quad \overrightarrow{BC}=3\mathbf{i}+\mathbf{k}$	**B1**	Finds direction vectors of **two** lines in the plane.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -1 & -2 \\ 1 & -1 & -1 \end{vmatrix} = \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}$	**M1**	Finds normal to the plane ABC by taking cross product of two of their vectors from above. Allow one numerical or sign error in cross product for M1.
		A1FT	Allow FT their vectors from above.
	$-(4)-4(3)+3(3)=-7 \Rightarrow -x-4y+3z=-7$	**M1**	Substitutes a correct point.
		A1	CAO
		5	

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Question	Answer	Marks	Guidance
5(b)	$\begin{pmatrix}2\\1\\3\end{pmatrix}\bullet\begin{pmatrix}-1\\-4\\3\end{pmatrix}=\sqrt{14}\sqrt{26}\cos\alpha \Rightarrow \cos\alpha=\frac{3}{\sqrt{14}\sqrt{26}}$	**M1**	Uses scalar product of $\overrightarrow{OD}$ and their normal. For M1 the scalar product and lengths must be evaluated but need not be simplified.
		A1FT	Lengths and scalar product simplified correctly.
	Acute angle between OD and Π is $90-\alpha=9.05^\circ$	**A1**	Mark final answer. Accept 9.0° 9.1° or 0.16 rad. But not 9°
		3	

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Question	Answer	Marks	Guidance
5(c)	**EITHER:** $\overrightarrow{CD}=\begin{pmatrix}2\\1\\3\end{pmatrix}-\begin{pmatrix}5\\2\\2\end{pmatrix}=\begin{pmatrix}-3\\-1\\1\end{pmatrix}$	**(B1**	
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & -1 & -2\\-3 & -1 & 1\end{vmatrix}=\begin{pmatrix}-3\\8\\-1\end{pmatrix}$	***M1**	Find common perpendicular of their CD and AB. Again, at most one error in cross product.
		A1	Correct answer
	$\frac{1}{\sqrt{74}}\left\lvert\begin{pmatrix}-1\\1\\1\end{pmatrix}\bullet\begin{pmatrix}-3\\8\\-1\end{pmatrix}\right\rvert=\frac{10}{\sqrt{74}}=1.16$	**DM1**	Uses formula for shortest distance. May use any vector joining a point of AB to a point of CD. For M1 the scalar product and length must be evaluated but need not be simplified.
		A1)	CAO Surd or decimal.
	OR: **Alternative method for 5(c)**		
	$\mathrm{r}=\begin{pmatrix}4\\3\\3\end{pmatrix}+\alpha\begin{pmatrix}-2\\-1\\-2\end{pmatrix}\quad \mathrm{r}=\begin{pmatrix}5\\2\\2\end{pmatrix}+\beta\begin{pmatrix}-3\\-1\\1\end{pmatrix}$	**(B1**	Write down a general point on each of AB and CD.
	$\begin{pmatrix}-1-2\alpha+3\beta\\1-\alpha+\beta\\1-2\alpha-\beta\end{pmatrix}$	**M1**	Finds vector joining them.

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Question	Answer	Marks	Guidance
5(c)	$-1+9\alpha-5\beta=0$ $3+5\alpha-11\beta=0$	**M1**	Scalar product of each line direction with this vector is zero. Both equations needed but can be unsimplified.
	$\alpha=\frac{13}{37}\quad\beta=\frac{16}{37}$	**A1**	Solves the equations.
	$\begin{pmatrix}-\frac{15}{37}\\ \frac{40}{37}\\ -\frac{5}{37}\end{pmatrix}$ has magnitude $\frac{10}{\sqrt{74}}=1.16$	**A1)**	CAO Surd or decimal.
		5	

Question	Answer	Marks	Guidance
6(a)	$x=0,\ x=1$	**B1**	Both vertical asymptotes.
	$y=1$	**B1**	Horizontal asymptote and no extra asymptotes.
		2	

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Question	Answer	Marks	Guidance
6(b)	$\frac{dy}{dx}=\frac{-2x^2-4x+2}{\left(2x^2-2x\right)^2}$	***M1**	Finds $\frac{dy}{dx}$. A method which clearly demonstrates the use of quotient rule, product rule or rearrangement must be seen.
	$x^2+2x-1=0$	**DM1**	Sets equal to 0 and forms quadratic equation.
	$(-2.4, 0.9)$, $(0.4, -1.9)$	**A1 A1**	Accept $\left(-1-\sqrt{2}, -\frac{1}{2}+\sqrt{2}\right)$, $\left(-1+\sqrt{2}, -\frac{1}{2}-\sqrt{2}\right)$ A1 A0 for correct x values but y values missing or incorrect.
		4	

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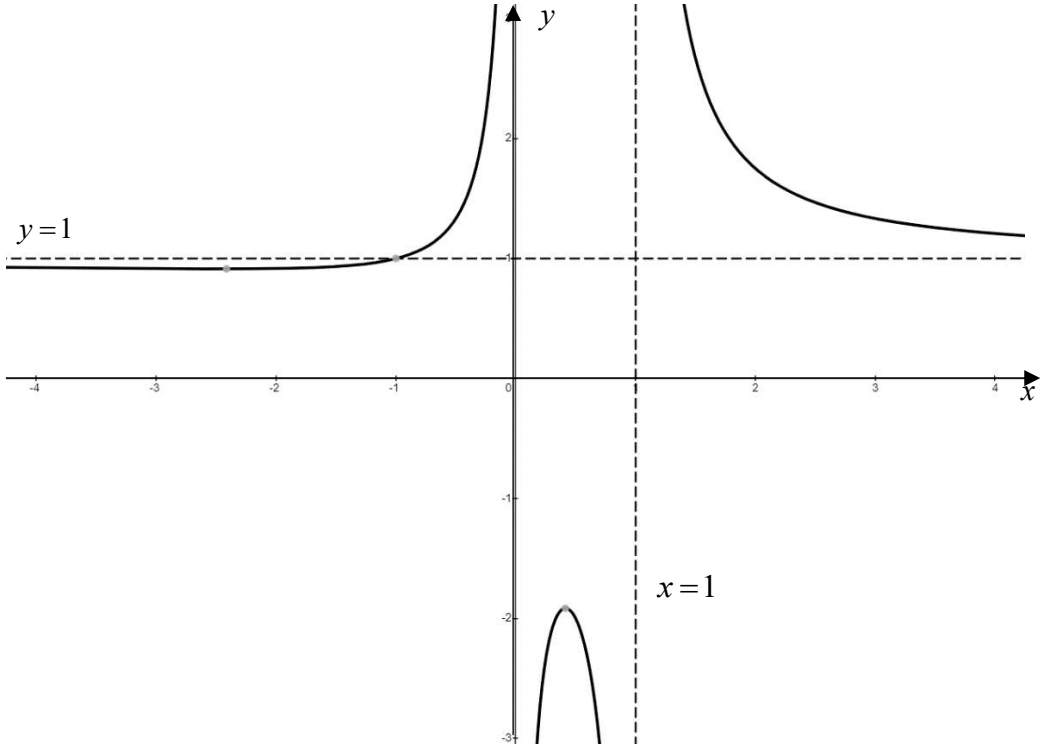
Question	Answer	Marks	Guidance
6(c)		**B1FT**	Axes and asymptotes labelled. Accept labelled by numerical intercepts with asymptotes. Allow FT from **(a)** for this B mark only.
		B1	LHS correct shape and position, crossing horizonal asymptote. The minimum should be indicated by the shape of the curve and the asymptotic approach. Do not expect accurate scale.
		B1	Both branches for $x > 0$ correct.
		3	

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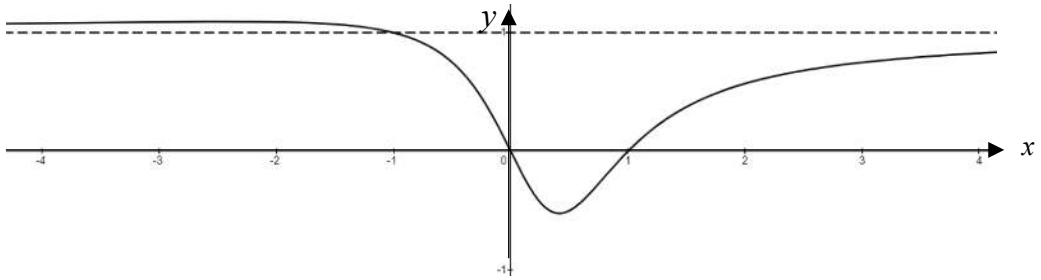
Question	Answer	Marks	Guidance
6(d)		**B1FT**	Must be a single curve from LHS to RHS of the page, with no breaks. Follow through the max / min for their graph for **(c)** as long as it has three branches. Numerical values need not be considered for this mark.
		B1	Correct shape and intersections with axis and relationship with asymptotes.
	$-1<x<0$	**B1**	
	$x>1$	**B1**	
		4	

Question	Answer	Marks	Guidance
7(a)	$\left(r^2\right)^{\frac{3}{2}}=(r\cos\theta)(r\sin\theta)$	**M1**	Uses at least two of $x^2+y^2=r^2$, $x=r\cos\theta$ and $y=r\sin\theta$.
	$r^2\left(r-\frac{1}{2}\sin 2\theta\right)=0$	**M1**	Uses correct identity. $\sin 2\theta=2\sin\theta\cos\theta$
	$[r\neq 0]\Rightarrow r=\frac{1}{2}\sin 2\theta$	**A1**	AG.
		3	

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Question	Answer	Marks	Guidance
7(b)	$\left(\frac{1}{2}\sin 2\theta\right)^2 = \dfrac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}$ Or $\frac{1}{2}\sin 2\theta = \left(\dfrac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}\right)^{\frac{1}{2}}$	**M1**	Sets r values equal.
	$\frac{1}{4}\sin^2 2\theta\left(\left(\theta - \frac{1}{4}\pi\right)^2 + 5\right) = 1 \Rightarrow \sin^2 2\theta\left(\left(\theta - \frac{1}{4}\pi\right)^2 + 5\right) = 4$	**A1**	AG. Intermediate working must be shown. We need to see e.g. ¼ or 2^2
.	$\sin^2(2\times 0.5)\left(\left(0.5 - \frac{1}{4}\pi\right)^2 + 5\right) - 4 = -0.4$ $\sin^2(2\times 0.6)\left(\left(0.6 - \frac{1}{4}\pi\right)^2 + 5\right) - 4 = 0.4$	**B1**	Shows sign change, calculates to at least 1dp. –0.401958833 0.373343553 No conclusion required. Allow 3.598 < 4, 4.3733>4
		3	

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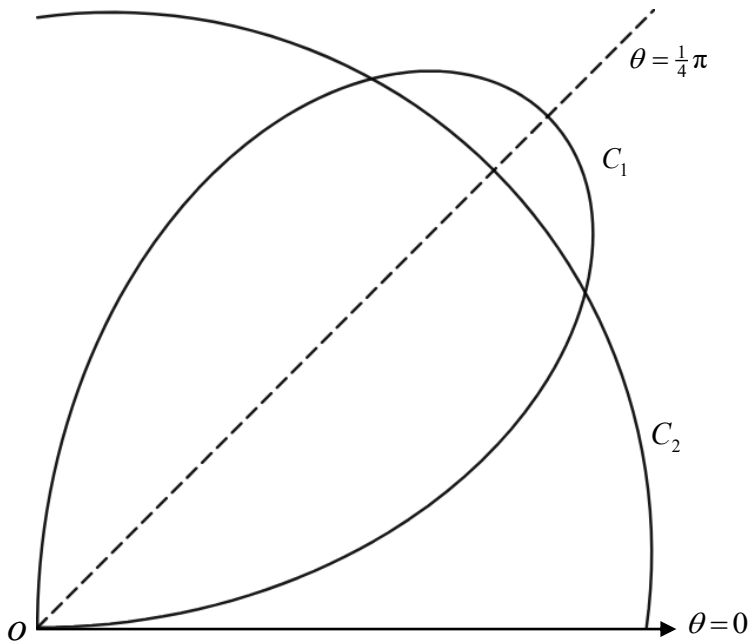
Question	Answer	Marks	Guidance
7(c)		**B1**	Initial line drawn and petal shape for C_1 in the first quadrant, starting and ending at the pole. Ignore graph in other quadrants for this B1.
		B1	Convex arc shape for C_2 with the pole on the concave side. Ignore graph in other quadrants for this B1.
		B1	Line of symmetry labelled on the diagram or equation given and sketch symmetrical about $\theta = \frac{1}{4}\pi$. Accept labelled as $\frac{1}{4}\pi$. Both curves labelled. Graph only in first quadrant.
		B1	C_1 Symmetrical, rounded end, apparently tangential to $\theta = 0$ and $\theta = \frac{\pi}{2}$ at the pole. C_2 Symmetrical, angle between curve and **positive** direction of $\theta = 0$ not more than $\frac{\pi}{2}$. A line drawn vertically from the start point of the arc would not intersect the petal. Graph only in first quadrant.
		4	

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Question	Answer	Marks	Guidance
7(d)	$\int_0^{\alpha} \sin^2 2\theta \, \mathrm{d}\theta = \frac{1}{2}\int_0^{\alpha} 1 - \cos 4\theta \, \mathrm{d}\theta = \frac{1}{2}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\alpha}$	**M1**	Integrates $\sin^2 2\theta$ using **correct** identity. Allow M1 if at most one numerical or sign error in the integration step.
		A1	(Seen somewhere, condone incorrect/missing limits.)
	$\int_{\alpha}^{\frac{1}{4}\pi} \frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5} \mathrm{d}\theta = \frac{1}{\sqrt{5}}\left[\tan^{-1}\left(\frac{\theta - \frac{1}{4}\pi}{\sqrt{5}}\right)\right]_{\alpha}^{\frac{1}{4}\pi}$	**M1**	Integrates $\frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}$. For M1 must have $p\tan^{-1} k\left(\theta - \frac{1}{4}\pi\right)$ where p and k are numerical.
		A1	A1 for fully correct. (Seen somewhere, condone incorrect/missing limits.)
	$\frac{1}{4}\int_0^{\alpha} \sin^2 2\theta \, \mathrm{d}\theta + \int_{\alpha}^{\frac{1}{4}\pi} \frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5} \mathrm{d}\theta$	**M1**	Forms area of R with correct limits. There are other possibilities.
	$= \frac{1}{8}\left(\alpha - \frac{1}{4}\sin 4\alpha\right) - \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{1}{\sqrt{5}}\left(\alpha - \frac{1}{4}\pi\right)\right)$ $= \frac{1}{8}\left(\alpha - \frac{1}{4}\sin 4\alpha\right) + \frac{1}{5}\sqrt{5}\tan^{-1}\left(\frac{1}{5}\sqrt{5}\left(\frac{1}{4}\pi - \alpha\right)\right)$	**A1**	AG. Use of limits seen and all previous marks must have been awarded.
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